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Supersonic Boundary-Layer Stability in a High-Area-Ratio Nozzle

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Previous studies found that the wall heat flux measured experimentally in the NASA Lewis high-area-ratio nozzle was not always described accurately by laminar or fully turbulent boundary-layer computations. These results suggest that the boundary layer undergoes transition. In the present paper, the e^N method was used to determine which operating conditions produced boundary-layer transition in the nozzle. The study verifies the accuracy of the e^N method for combusting flows. If transition resulted, the stability analysis provided the location of transition onset as well as the structure and wavelength of the dominant instability. Tollmien-Schlichting waves and Taylor-Görtler vortices were considered as possible disturbance structures which trigger transition. When the chamber pressure was 2482 kPa, the stability analysis indicated that transition did not occur in the nozzle. The analysis correctly determined that a laminar boundary-layer computation accurately predicts the experimental wall heat flux. For chamber pressures of 4523 kPa and above, it was found that Taylor-Görtler vortices produced transition in the nozzle. At these conditions, the laminar boundary-layer computation did not accurately predict the wall heat flux. The stability analysis can be used to establish the nozzle conditions which will be accurately predicted by a laminar boundary-layer computation.

Nomenclature

a = amplification ratio
C_p = specific heat at constant pressure
k = thermal conductivity
m = number of Taylor-Görtler vortices around the nozzle perimeter
N = exponential factor for the amplification ratio
P_c = chamber pressure
$r(x)$ = nozzle radius
x = axial distance from the nozzle throat
α = amplification rate
δ = boundary-layer thickness
γ = ratio of specific heats
λ = wavelength of Görtler vortices
ξ = distance from the nozzle throat along the nozzle surface
μ = dynamic viscosity

Subscripts

n = at neutral stability
t = at transition

I. Introduction

To design a rocket nozzle, it is necessary to determine the features of the gas flow through the nozzle including the characteristics of the nozzle wall boundary layer. The nature of the nozzle wall boundary layer determines the wall heat flux which directly dictates the cooling-system requirements. At a given streamwise location on the nozzle wall, a laminar boundary layer exhibits a lower wall heat flux than a turbulent boundary layer. If it is not known whether boundary-layer transition has occurred, the cooling system must be designed for the worst case, the high-heat-flux turbulent boundary layer. Accurate prediction of the boundary-layer transition location is needed to prevent overdesign of the cooling system.

Although many characteristics of supersonic flow are well established, the transition of a supersonic laminar boundary layer is still poorly understood. Experimental measurements of transition are difficult in rocket nozzle applications since the flow temperature exceeds 2300°C. Under these conditions, only the nozzle thrust and the temperature distribution along the external nozzle surface can be measured presently. Although the rate of increasing wall temperature can be used to determine the internal wall heat flux, it cannot be used to deduce the flow velocity or temperature fields.

To understand nozzle flows better, numerical investigations are employed. These computations deliver complete velocity and temperature fields within the nozzle. As a check, the results can be compared with the experimental thrust and wall heat flux measurements. If the experiment and the computation yield similar values for these important parameters, the flow conditions found using the computational method can be considered to describe the flow actually present in the experimental facility. The computation can then deliver additional information which cannot be accurately measured. Once verified, the successful numerical scheme can be used as a design tool to predict the performance of candidate nozzle designs with only limited experimental checks.

Typically, a nozzle flow computation assumes the presence of either a laminar or fully turbulent boundary layer; the transition region is often not accurately located. By computing results for both the completely laminar boundary layer and the completely turbulent boundary layer, the minimum and maximum possible wall heat flux at a particular location is determined for a specified geometry. Often experimental heat flux measurements are between these two limits, leaving the nature of the boundary layer unknown. The boundary layer may have transitioned from laminar to turbulent or three-dimensional structures may be present. A boundary-layer stability analysis can be used to locate the onset of transition or the growth of three-dimensional structures in a laminar boundary layer.

On locating the onset of transition, transition models can be employed to describe the transition region accurately. Narasimha[1,2] details the methods commonly used to model the transition region. During the early stages of transition, Narasimha[2] observes that linear stability theories accurately describe the development of disturbances. In the later stages

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of transition, most models use an intermittency parameter to describe the occurrence of spots of turbulence within an otherwise laminar boundary layer. There is only a small region where neither the linear stability analysis nor the intermittency model accurately describes the transition. In this region, nonlinear stability analysis is required. Narasimha comments that accurate models for the transition region have been developed and the primary challenge is in accurately locating the onset of transition.

In this paper, a linear stability analysis is used to understand the development of a supersonic boundary layer in a rocket nozzle. The study focuses on experiments in the NASA Lewis high-area-ratio nozzle that produced wall heat flux values between those predicted for the extremes of laminar and fully turbulent boundary layers. Through a stability analysis, the location of transition is estimated and the structure of the dominant instability is characterized.

Stability analysis and e^N methods have been used by many investigators to study both incompressible and compressible flows. In the present study, these methods will be used to study a combusting flow in a rocket nozzle. The authors are not aware of other research that has applied the stability analysis to a combusting flow.

II. Background

A. NASA Lewis High-Area-Ratio Supersonic Nozzle Experiment

The performance characteristics of the high-area-ratio nozzle at the NASA Lewis Research Center have been investigated in recent years. This nozzle is a low-thrust design for space applications such as for orbital transfer vehicles. The nozzle has a throat diameter of 2.54 cm and an expansion area ratio of 1030:1. The geometry of the nozzle is shown in Fig. 1. Firings with gaseous H_2 and gaseous O_2 as propellants have been performed at high-altitude conditions to measure the thrust of the nozzle. Temperature measurements were made on the outer surface of the nozzle, and the rate at which the temperature increases was employed to determine the wall heat flux. Because the temperature of the nozzle gas flow was over 2300°C, no measurements such as velocity and temperature within the nozzle were possible. Although the available data can be used to place lower bounds on the required nozzle cooling system capacity, they do not provide means for an informed description of the flow characteristics in the nozzle. This capability is needed to develop improved tools for the design of economical cooling systems for future nozzles.

Early studies of the high-area-ratio nozzle by Pavli et al.,[3] Smith et al.,[4] and Kacynski et al.[5] tested low chamber pressure cases. The chamber pressure for all tests was near 2482 kPa and the propellant mixture ratio was varied from 2.69 to 6.03. The thrust measured in the nozzle experiments ranged from 2.22 to 2.45 kN. Smith et al. and Kacynski et al. also computationally studied the nozzle flow using the two-dimensional

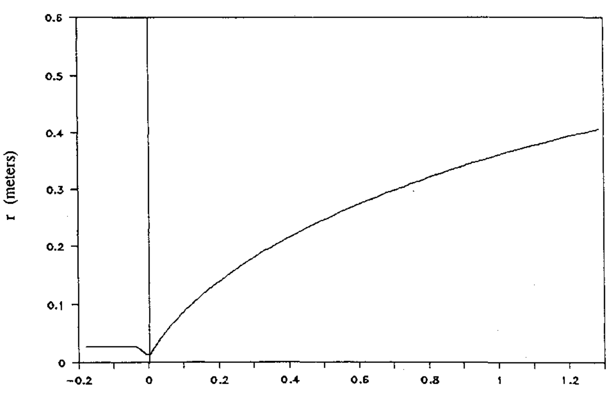

Fig. 1 High-area-ratio nozzle geometry.

kinetics (TDK) computer analysis program[6] under axisymmetric conditions. Since the TDK program computes the nozzle performance for completely efficient combustion, the experimental results were modified to reflect the thrust and wall heat flux expected if the nozzle combustion was completely efficient. For each case two computations were performed, one with a laminar boundary layer and the other with a fully turbulent boundary layer. It was found that all experimental wall heat flux and thrust measurements were accurately predicted by the laminar boundary-layer computations. This indicates that the boundary layer did not undergo transition at low chamber pressures.

Higher chamber pressure cases were studied experimentally and numerically by Smith.[7] She varied the chamber pressure from 2482 to 6922 kPa and the nozzle thrust ranged from 2.22 to 5.34 kN. Smith found that her laminar computations compared well with the adjusted experimental results when the chamber pressure was low. When the chamber pressure was increased to 4523 kPa, the experimental heat flux fell between the computed values for a laminar and turbulent boundary layer. The higher chamber pressure cases up to 6922 kPa also produced a measured wall heat flux that was between the laminar and turbulent boundary-layer predictions. The characteristics of the boundary layer were not correctly described by the fully laminar or fully turbulent boundary-layer computation.

Under these conditions, the nature of the boundary layer cannot be deduced from the data. The boundary layer may be altered by an instability inducing transition to turbulence or by a stable three-dimensional structure present in the boundary layer. An inaccurate description of the experimental conditions including flow kinetics, inflow conditions, and wall temperature could also cause the computation to be in error. The source of the discrepancy cannot be determined from the computational results. Since rocket nozzles typically operate at chamber pressures of 6922 kPa and above, it is important to understand the mechanism which causes an enhanced heat flux under these conditions.

B. Boundary-Layer Stability Analysis

To determine the extent of three-dimensional structure growth in an otherwise two-dimensional laminar boundary layer, a stability analysis can be used. For the development of a boundary layer on a concave surface, two different structures can trigger transition of the boundary layer: Tollmien-Schlichting waves and Taylor-Görtler vortices. Tollmien-Schlichting waves are planar waves that travel in the streamwise and spanwise directions. Taylor-Görtler vortices are an array of steady longitudinal vortices embedded in the boundary layer. This instability can develop in boundary layers on concave surfaces. When analyzing the stability of a laminar boundary layer, a spectrum of possible wavelengths is considered for each instability structure to determine the wavelength and structure of the instability that grows most rapidly. If the disturbance grows to a specified amplitude, the disturbance is of sufficient strength to cause the two-dimensional laminar boundary layer to become three dimensional. The location where the instability reaches the critical amplitude can be identified as the onset of transition from a two-dimensional to a three-dimensional boundary layer. The three-dimensional boundary layer containing coherent structures may be a stable flow or may lead to transition to a two-dimensional turbulent boundary layer.

Stability studies use perturbation techniques to determine the growth rate of a disturbance with a given frequency and structure. In the stability analysis, the velocity field is described by a steady laminar component and a perturbation component. The mean flow is assumed to be locally parallel and is only a function of the normal coordinate direction. For flows with favorable pressure gradients, Saric and Nayfeh[8] have found that nonparallel effects are insignificant. Each possible instability is analyzed separately; the interaction between different disturbances is not considered. This is an accurate assumption during the onset of transition where the instabilities are weak. Details of the numerical methods used in the stability analysis can be found in Malik.[9]

The amplification ratio a can be used to determine which disturbance has grown most significantly throughout the boundary-layer development. It is defined as the ratio of the amplitude of a disturbance to its amplitude at neutral stability, the location where the disturbance becomes unstable. The amplification ratio is found by integrating the amplification rate along the nozzle surface from the location of neutral stability.

$$a = \exp\left(- \int_{\xi_n}^{\xi} \alpha \, d\xi^* \right) \tag{1}$$

The Tollmien-Schlichting wave is described by the wavelength of the instability. When tracking the growth of a Taylor-Görtler vortex in an axisymmetric geometry, the disturbance is defined in terms of m, the number of vortices along the surface. The varying perimeter of the nozzle changes the wavelength of a particular Taylor-Görtler instability during the downstream development.

The disturbance which has the largest amplification ratio will initiate transition of the laminar boundary layer. Smith[10] found that transition typically occurs where the amplification ratio reaches e^9 to e^{11} when the freestream turbulence is low. This criterion has been found to give an accurate prediction of the transition point when either Tollmien-Schlichting or Taylor-Görtler instabilities cause transition of compressible or incompressible boundary layers. This method for predicting the transition location is known as the e^N method.

Recent transition studies which successfully used the e^N method for low turbulence freestream conditions include a Tollmien-Schlichting transition study by Horstmann et al.[11] and a Taylor-Görtler transition study by Chen et al.[12] Chen et al. investigated the transition of the boundary layer in the diverging nozzle of a supersonic wind tunnel. Tollmien-Schlichting and Taylor-Görtler instabilities were considered along the curved wall. From a boundary-layer stability analysis, they found that Taylor-Görtler instabilities grew more rapidly in the supersonic nozzle and induced the transition of the laminar boundary layer on the wind-tunnel walls. Transition in the experimental facility occurred at the location where the amplification ratio had a value of e^9 to e^{11}. They suggest an amplification ratio of $e^{9.2}$ (N factor of 9.2) as a design criterion for transition.

When the freestream turbulence level is high, the amplification ratio at transition can drop significantly. Since the initial amplitude of the disturbance is inherently larger when freestream turbulence is present, the amplification ratio required to produce transition will therefore be reduced. Mack[13] found that the N factor dropped from 8.1 to 2.6 when the freestream turbulence was increased from 0.1 to 1% of the freestream velocity. The freestream turbulence level, therefore, must be known before an accurate boundary-layer transition criterion can be established.

III. Results

A. Axisymmetric Boundary Layer

The laminar boundary-layer solution needed for the stability analysis was determined using TDK for an axisymmetric geometry. Five chamber pressure conditions within the NASA Lewis high-area-ratio nozzle (Smith[7]) were studied. The experimental wall temperature distribution was specified within the nozzle skirt. Experimental surface temperature measurements were not possible in the combustion chamber and transonic flow regions due to the experimental facility. In this region adiabatic wall conditions were specified. This approximation was considered adequate since a 30% decrease in the surface temperature was found to change the downstream wall heat flux by less than 5%.

The present work differs from that of Smith by accounting for the combustion inefficiency within the TDK computation. For each chamber pressure case, TDK was first run to determine the chamber temperature for an efficient combustion process. This computation yielded a characteristic exhaust velocity which was greater than the experimental value. To simulate the combustion inefficiency of the experiment, the chamber temperature was lowered in the TDK computation until the characteristic exhaust velocity of the experiment was matched. Since the nozzle thrust produced during inefficient combustion was predicted to within 6%, the nozzle combustion conditions were considered accurately described. This method allows for a direct comparison between the computational and experimental wall heat flux distributions.

The TDK computation used the method of characteristics in the inviscid flow region and a boundary-layer computation in the viscous region. The boundary-layer computation applied 45 mesh points across the thickness of the boundary layer. Grid stretching was used to resolve the velocity gradients near the nozzle wall. The computation was initiated in the combustion chamber and was marched 1000 streamwise locations to the end of the nozzle. The streamwise distribution of points was such that the wall gradients of temperature and velocity were properly resolved. At every streamwise location, the thickness of the boundary layer was determined. Additional grid points were added in the normal direction if the edge of the boundary layer approached the edge of the computational domain.

Figure 2 shows the TDK wall heat flux for fully laminar and fully turbulent boundary-layer computations at three chamber pressures tested by Smith. Included in the figures are the experimental heat flux values (not modified). As Smith had found, the laminar boundary-layer computation accurately described the experimental wall heat flux at the lowest chamber pressure, Fig. 2a. The accuracy of the laminar boundary-layer computation suggests that transition does not occur. It will be seen later that the stability analysis confirms the absence of transition at 2482 kPa chamber pressure.

At 4523 kPa shown in Fig. 2b, the laminar boundary layer computation accurately reproduces the experimental wall heat flux for much of the nozzle length. The experimental results diverge from the laminar computational values near the exit of the nozzle. The stability analysis results presented in Sec. III.B confirm that transition occurs in this region of the nozzle. Near the nozzle throat, it is observed that the experimental wall heat flux is not accurately described by the laminar boundary-layer computation. It is conjectured that this is due to an inaccurate estimation of the surface temperature conditions at the throat of the nozzle. Since the throat was made of copper and water-cooled whereas the remainder of the nozzle was steel and uncooled, estimations of the throat surface temperature may be in error by more than 30%.

For chamber pressures above 4523 kPa, the experimental heat flux lies between the heat flux determined by the laminar boundary-layer computation and turbulent computation as seen in Fig. 2c. As the chamber pressure is increased, the experimental heat flux moves further from the laminar boundary-layer results and closer to the fully turbulent boundary-layer prediction. The experimental heat flux, however, never reaches the turbulent boundary-layer values. At high chamber pressures, it appears that transition has occurred during the boundary-layer development. The stability analysis at these chamber pressures will confirm the presence of boundary-layer transition.

B. Boundary-Layer Stability Analysis

The linear spatial stability program developed by Malik[14] was used to study the growth of three-dimensional structures within the laminar axisymmetric boundary layer. At each chamber pressure, the laminar boundary-layer development determined by TDK was used as the mean flow solution. Tollmien-Schlichting waves and Taylor-Görtler vortices were

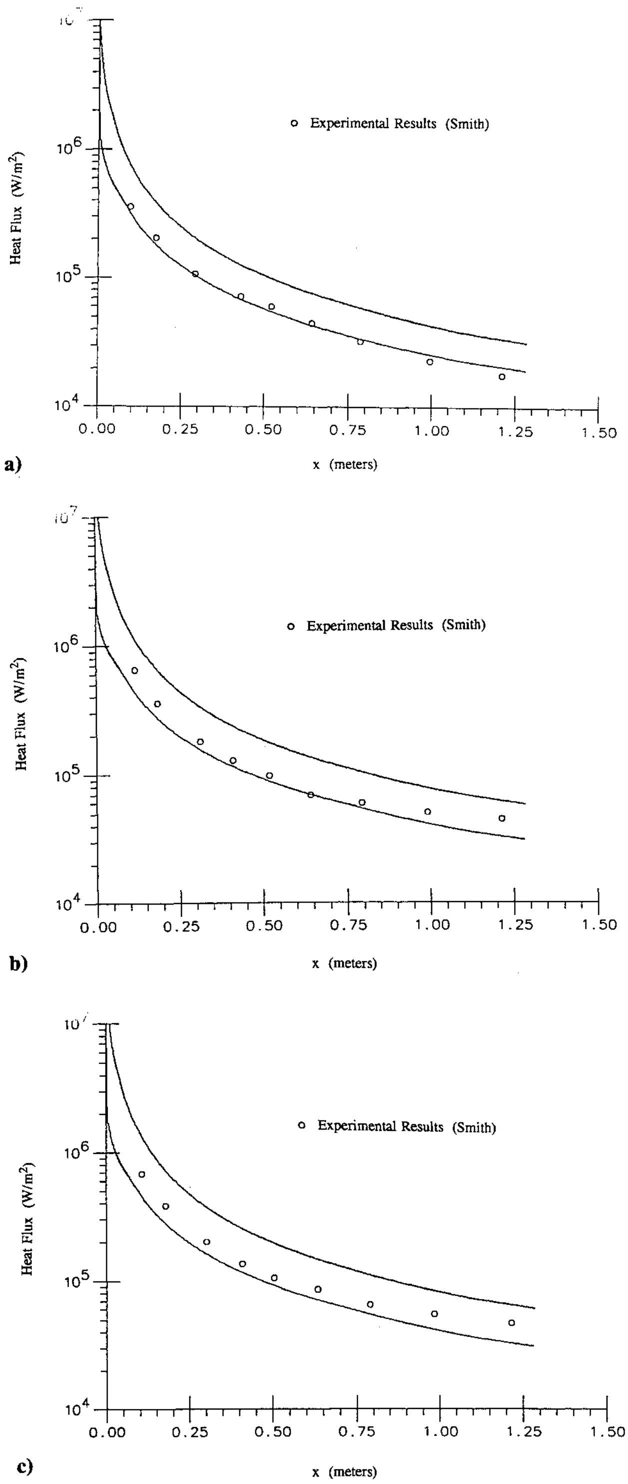

Fig. 2 Experimental and predicted wall heat flux. Computational heat flux results were modified to represent the inefficient combustion conditions of the experiment: a) P_c = 2482 kPa; b) P_c = 4523 kPa; and c) P_c = 6922 kPa.

considered as possible instabilities. The wavelength of the dominant instability was identified using the e^N method. If the N factor reached the critical N factor, the boundary layer was considered transitional. The location where the dominant instability reached the critical N factor was denoted as the onset of transition.

To integrate the amplification rate of a Taylor-Görtler structure in the axisymmetric nozzle geometry, a constant number of vortices around the perimeter of the nozzle must be retained throughout the boundary-layer development. Since the perimeter of the nozzle changes in the streamwise direction, the wavelength of the instability does not remain constant. The program e^{Malik} was modified so that the vortex number m remains constant during the streamwise integration of the amplification rate. The program was also modified so that the gas properties (γ, μ, k, C_p) and their derivatives with respect to temperature could be specified in the stability analanalysis using the results from TDK.

The program e^{Malik} includes a global and local eigenvalue search. The implicit global eigenvalue search neglects the streamwise diffusion terms when locating an approximate eigenvalue. Streamwise diffusion terms are included in the explicit local search which employs fourth-order differencing and Newton's iteration method. The local search is initiated using a guess of the eigenvalue solution from the local search at the previous streamwise location. If the local search does not converge or the local solution is not known at the previous streamwise location, the global search is employed and the results are used as a new initial guess in the local eigenvalue search. From the eigenvalue solution, the amplification rate of a particular instability is determined.

When defining the mean velocity profiles to be used in the stability analysis, boundary-layer data from TDK were redistributed into an exponential distribution using a spline fit. The global search typically used 81 grid points whereas the local search used up to 144 grid points across the boundary layer. The grid resolution was adjusted to minimize the number of grid points while insuring that the amplification rate was unaffected. Profiles were saved at 60 streamwise locations. The streamwise locations were positioned closer together in regions where the nozzle surface curvature was high and, therefore, the amplification factor changed rapidly. From the stability analysis, the amplification rate was determined at each streamwise location. The N factor was then obtained from the integration of the amplification rate. Comparisons of the present results with select tests using more streamwise locations indicate an error of no more than 5% in the N factor when 60 streamwise locations are used.

Two values of the N factor (8.5 and 9) were tested as transition criteria. An N factor of 9 is commonly defined as the critical N factor when the freestream turbulence level is low. It is expected that the actual N factor is below the theoretical value of 9 since freestream unsteadiness is inherently present in the internal nozzle flow since upstream conditions cannot be completely steady. If freestream turbulence exists, the critical N factor will be lower and transition will occur earlier. Although the combustion chamber flow will be highly turbulent, the rapidly accelerated freestream in the nozzle will reduce the turbulence level. It would be expected, therefore, that the critical N factor in the high-area-ratio nozzle would be only slightly lower than the steady freestream transition criterion. An N factor of 8.5 was used as an estimate for the critical N factor when the effects of freestream turbulence were considered. This N factor corresponds to the N factor at the experimentally measured transition location when P_c = 4523 kPa.

A stability analysis of Tollmien-Schlichting waves was conducted at chamber pressures of 2482 and 6922 kPa. At 2482 kPa, it was found that the frequency of the dominant Tollmien-Schlichting wave instability was 9 kHz. The instability reached a maximum N factor of 1.42 at the nozzle exit and had a wave angle of 75.8 deg. The instability was third mode, identified by the eigenvalue solution of the pressure perturbations twice crossing a value of zero before asymptoting to zero outside the boundary layer. At 6922 kPa, the dominant Tollmien-Schlichting wave instability grew slower and reached a maximum N factor which was 1.01. The frequency of the dominant Tollmien-Schlichting wave was 22 kHz at a wave angle of 0.0 deg. The dominant instability was a second-mode disturbance; the eigenvalue solution of the pressure perturbation crosses zero once before asymptoting to zero in the freestream. In the supersonic nozzle, the accelerating free-

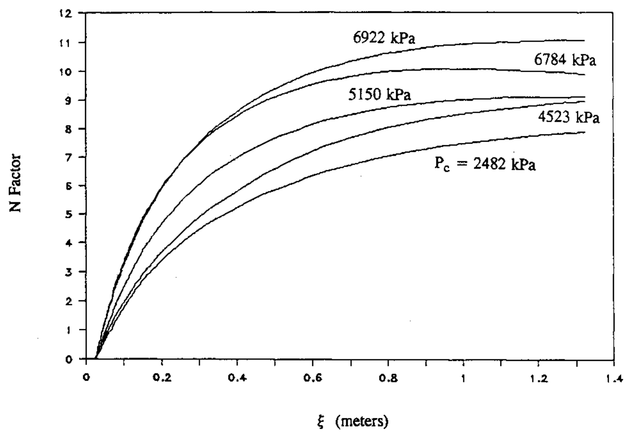

Fig. 3 N factor variation along the nozzle surface for the dominant vortex number at each chamber pressure.

stream causes the dominant Tollmien-Schlichting waves to grow only slowly through the boundary-layer development. These results indicate that Tollmien-Schlichting waves are not responsible for the transition of the laminar boundary layer.

For all chamber pressures, the Taylor-Görtler vortices were amplified more rapidly than the Tollmien-Schlichting waves. N factor curves are shown in Fig. 3 for the dominant Taylor-Görtler instability at each chamber pressure. Since the amplification factor was integrated along the surface of the nozzle, the N factor distribution is plotted against ξ, the distance from the nozzle throat as measured along the surface of the nozzle. The axial distance from the nozzle throat x can be related to the surface distance ξ using the approximation:

$$\xi \approx 1.034x + 0.374 \text{ m} \tag{2}$$

This approximation has an accuracy within 0.4% from $12 < x < 37$. In all cases, the Taylor-Görtler instability grows rapidly near the throat where the wall first becomes concave and the curvature is maximum. Further downstream, the wall curvature is reduced and the Taylor-Görtler vortices grow at a slower rate.

Table 1 summarizes the maximum N factor found from the Taylor-Görtler stability analysis at the five chamber pressures tested. It can be seen that the maximum N factor found within the nozzle increases as the chamber pressure increases. The number of vortices present around the perimeter of the nozzle m is listed for the dominant Taylor-Görtler instability. As the chamber pressure is increased, the dominant instability contains more vortices around the nozzle surface.

At 2482 kPa, the Taylor-Görtler vortices reach a maximum N factor of 7.89. The stability analysis indicates that the growth of the instability is not strong enough to cause transition and the boundary layer will remain laminar at the lowest chamber pressure condition. The flow characteristics, including the wall heat flux, should be accurately predicted by a laminar boundary-layer computation. It can be seen in Fig. 2a that the laminar boundary-layer computation accurately described the experimental heat flux. The stability analysis correctly establishes the conditions where the boundary layer remains laminar. The wall heat flux can then be predicted accurately by a laminar boundary-layer computation.

The stability analysis for the 4523 kPa chamber pressure case produced a maximum N factor of 8.96 at the exit to the nozzle. If the nozzle freestream velocity was steady, the boundary layer would be considered as laminar through the entire nozzle. If the transition criterion accounts for the presence of freestream turbulence, the nozzle boundary layer will transition at $x = 92.8$ cm.

At chamber pressures of 5150 kPa and higher, the N factor exceeds 9.0, indicating a transition of the boundary layer when either critical N factor is used as the transition criterion. The stability analysis reveals that the boundary layer will transition and a laminar boundary-layer computation will not accurately reproduce the experimental wall heat flux. This finding is consistent with the preceding result, that the heat flux from the laminar boundary-layer computations using TDK duplicated the experimental heat flux only at 2482 kPa.

Table 2 gives the wall location and axial location where the N factor reaches 8.5 and 9, noted by the subscripts 8.5 and 9. The boundary layer is considered transitional at the location where the N factor reaches the transition criterion. At the lowest chamber pressure, the N factor does not reach 8.5 and the boundary layer remains laminar. All higher chamber pressure conditions result in a transition of the laminar boundary layer. As the chamber pressure was increased in a series of cases, the predicted transition location moved upstream.

At $P_c = 6922$ kPa, the instability containing 390 vortices around the nozzle perimeter reached a value of $N = 8.5$ furthest upstream. Note however that the instability containing $m = 350$ vortices reached the highest N factor. For each chamber pressure condition, instabilities having similar vortex numbers grew at similar rates and the N factor curves often crossed.

The stability analysis of Chen et al.[12] found many boundary-layer characteristics which were similar to the present investigation. First, in both studies the most unstable Tollmien-Schlichting waves grew only slowly within the accelerating region of the wind-tunnel nozzle. In the study of Chen et al., the maximum N factor for Tollmien-Schlichting waves was 1. Second, Taylor-Görtler vortices were more unstable than Tollmien-Schlichting waves. Chen et al. found that the Taylor-Görtler vortices were responsible for transition and reached an N factor between 9 and 11 at the transition location measured experimentally. Lastly, for the dominant disturbance, the number of vortices around the perimeter was similar for the axisymmetric wind tunnel and the present rocket nozzle.

The shape of the Taylor-Görtler vortices changes as the disturbance grows in the downstream direction. The height of the vortices λ_y can be compared with the spanwise wavelength λ_z to indicate the shape of the vortices. λ_y is approximated as twice the distance to the point at which the spanwise velocity perturbation crosses a value of zero. The aspect ratio λ_y/λ_z is shown in Fig. 4a for the dominant Taylor-Görtler instability at $P_c = 6922$ kPa. The vortices are initially wider than tall. Downstream the vortices proceed to become more circular in shape and then become much taller than wide. In Fig. 4b, the ratio of λ_y to the boundary-layer thickness δ reveals the relative size of the vortices to the boundary layer. The height of the vortices increases downstream with respect to the boundary-layer thickness. Near the nozzle throat, the approximate vortex height is 30% of the boundary-layer thickness. This indicates that most of the boundary layer remains two-dimensional despite the presence of a three-dimensional struc-

Table 1 Maximum Görtler N factors

P_c, kPa	m	N_{max}
2482	158	7.89
4523	200	8.96
5150	280	9.12
6784	369	10.06
6922	350	11.05

Table 2 Predicted transition location using N_t = 8.5 and 9.0

P_c, kPa	m	$\xi_{t8.5}$, cm	$x_{t8.5}$, cm	ξ_{t9}, cm	x_{t9}, cm
2482	None	None	None	None	None
4523	200	99.6	92.8	None	None
5150	280	70.7	64.6	99.7	92.9
6784	369	41.0	36.1	48.6	43.3
6922	390	37.2	32.6	43.3	38.2

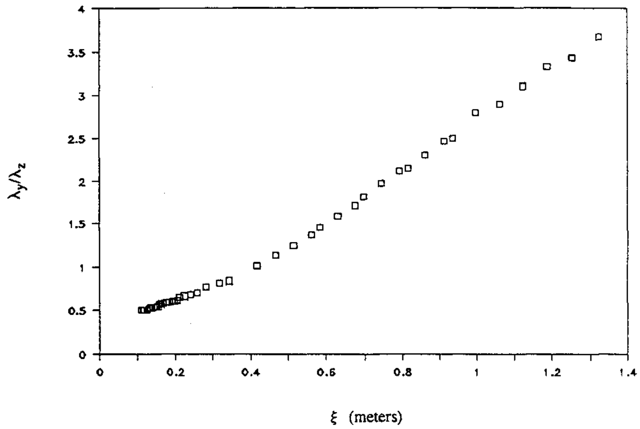

Fig. 4a λ_y/λ_z for P_c = 6922 kPa and m = 355.

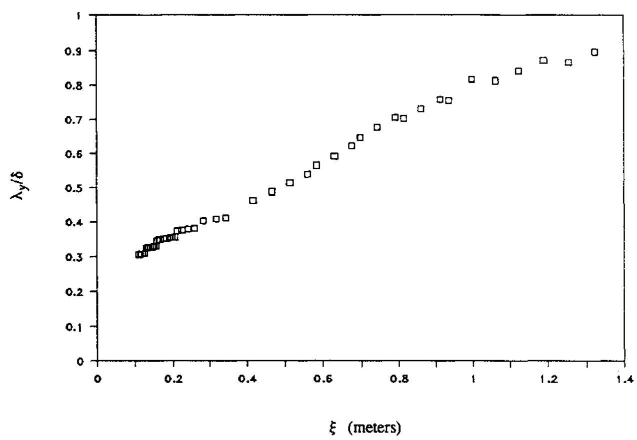

Fig. 4b λ_y/δ for P_c = 6922 kPa and m = 355.

ture near the wall. Since the vortex does not effectively entrain fluid from the freestream, it is expected that the vortex structure will not significantly enhance the wall heat flux. At the exit of the nozzle, the dominant Taylor-Görtler vortices reach 90% of the boundary-layer thickness. The entire boundary layer is modified by the three-dimensional structure which will cause an entrainment of freestream fluid. This will cause an enhancement of the wall heat flux in the region near the nozzle exit.

Further details of the boundary-layer stability study within the NASA Lewis high-area ratio nozzle can be found in Dagher.[15]

IV. Conclusion

The boundary-layer stability within the high-area-ratio nozzle at NASA Lewis was examined at five chamber-pressure conditions using the e^N method. Transition was considered to occur at the location where the amplification ratio reached $e^{8.5}$. Tollmien-Schlichting waves and Taylor-Görtler vortices were considered as possible instability structures.

For all nozzle conditions tested, Taylor-Görtler vortices grew more rapidly than Tollmien-Schlichting waves. For the 2482 kPa chamber pressure conditions, no disturbance reached an amplification ratio of $e^{8.5}$, indicating that transition was not expected at the lowest chamber pressure. The experimental heat flux was described accurately by a laminar boundary-layer computation thus confirming the laminar nature of the nozzle boundary-layer flow. At higher chamber pressures, the stability analysis indicated that boundary-layer transition had occurred. Laminar boundary-layer computations run at the higher chamber pressure conditions underpredicted the experimental wall heat flux whereas a fully turbulent boundary-layer computation overpredicted the experimental values. The boundary-layer computations confirmed the likelihood of transition within the nozzle.

As the chamber pressure was increased in a series of cases, the transition point occurred further upstream. The number of vortices contained in the dominant instability also increased with chamber pressure. This study demonstrates the accuracy of the e^N method for combusting flows.

Acknowledgments

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